**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS: B. (0.2676)

As in the question The service manager plans to have work begin on the transmission of customer’s car 10 minutes after the car is dropped off so the servicing work will begin shortly after the car is dropped off

Which will become

new mean time = 55min &

standard deviation=8min

**[Refer IPYNB]**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS:

1. False,

Mean = 38, Standard Deviation = 6

**[Refer IPYNB for better Explanation]**

1. True,

**[Refer IPYNB for better Explanation]**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS:

*X1* ~ *N*(μ, σ2)

*Mean: E(X1) =* μ

*Variance: var(X1) =* σ2

*X*2 ~ *N*(μ, σ2)

*Mean: E(X2) =* μ

*Variance: var(X2) =* σ2

*Difference between 2X1 and X1 + X2:*

1. *2X1*

*Mean: E(2X1) =2 x E(X1) = 2 x* μ

*Variance: var(2X1) =(22) x Var(X1) = 4 x* σ2

So, 2X1 is a normal random variable with mean 2μ and variance *4*σ2.

1. X1 + X2

Mean: E(*X*1 + *X*2) = E(*X*1) + E(*X*2) = *2 x* μ

Variance: Since X1 and X2 are independent, the variance of their sum is the sum of their variances.

Var(*X*1 + *X*2) = var(*X*1) + Var(*X*2) = 2 σ2

So, *X*1 + *X*2 is a normal random variable with mean 2μ and variance *2*σ2.

By seeing the above derivations

While both *2X1  and X*1 + *X*2  have the same mean, their variances are different due to the different properties of multiplication and addition. 2X1 has a larger spread compared to *X*1 + *X*2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

ANS: D.

Mean = 100, Std = 20, Confidence Interval = 0.99

By using **stats.norm.interval(confidenceInterval, mean,std)** we can get the a & b values

**[Refer IPYNB]**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

ANS:

Mean = (5+7) \*45 = 540(million)

Standard Deviation = sqrt((9)+(16)) = 5

= 5 \* 45 = 225(million)

1. Rupee range is (99 to 980) Million Rupees **[Refer IPYNB]**
2. To find the 5th percentile, we use the formula of z table, 5% = -1.645

X = 540 + (-1.645) \* (225)

X = 170

The 5th Percentile of profit for the company is 17 Crore Rupees (or) 170 Million Rupees.

1. The Division 2 (Profit2 ~ N(7,43)) has a larger Probability of making a loss in a given year **[Refer IPYNB]**